

Analysis on The Origin, Development and Contribution of Ge'ez Numerals to The Modern Mathematics Education in Ethiopia: Status, Challenges and Future Opportunities.

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Abstract:

From the Ethiopian ancient history, it is believed that among the first peoples to inhabit was the Ge'ez speaking agrarians who settled in the Tigrayan highlands (Northern Ethiopia) around 2000BC. The inland Axum Kingdom was founded by Menilik I, a descendent of King Solomon and the Queen of Sheba, after the fall of the Da'amat Kingdom. Associated with the letters in the Ethiopian alphabet starting from ancient times, there are the Ge'ez numerals also called Ethiopic numerals. This study, therefore, purely used qualitative data obtained from interviewees and document analysis. The data obtained are narrated and organized. Patterns are identified and categorized. Finally, interpretations and implications of findings are included in the analysis. Findings indicated that there is a need for modification of the Ge'ez numbers by removing the bars above and below the original skeleton of the numbers so that doing mathematics with them becomes easier. Beyond this modification, evidence indicated that the Ge'ez numbers cannot be used for doing mathematical operations without the introduction of Ge'ez zero. Without the Ge'ez zero, it is impossible to deal with place value, decimal, and fractions, as well as with the common operations (addition, subtraction, multiplication, and division) and higher mathematics in general. Finally, it is the recommendation of this study to work towards awareness creation and consensus building about the common understanding and misunderstanding of Ge'ez numeration system at all levels as well as introducing this numeration system in to the school system.

Key words: Ethiopic Numbers, Ge'ez Alphabet, Ge'ez Numerals, Ge'ez Numeration System

1. Introduction

Ethiopia is a landlocked country in Eastern Africa located on the Horn of Africa. It is bordered by Eritrea to the north, Djibouti to the east, Kenya to the south, and the Sudan and South Sudan to the west. The size of the country is around 1,133,882 square kilometers.

As pointed out in MOE –ESDP V. (2015) document, Ethiopia is a big, diverse country with a population of over 115 million, in more than 90 ethnic and linguistic groups with a population density of 86 people per square km. Four in five of the population live in the highlands, temperate parts of the country. The remaining one in five of the population, mostly pastoral and agro-pastoral groups, live in the lowlands that cover 60% of the country's land area. As indicated in Central Statistical Agency (CSA, 1994, 2008) report of the country, Ethiopia has an annual nationwide average population growth rate of 2.6% (2.5% for Tigray region) from the year 1994 to 2007. Ethiopia is a mountainous region in the East and West highlands divided by the Great Rift Valley. It is one of the oldest kingdoms in the world, a place where the first mankind is found in the Afar region (within the Great Rift Valley). It is one of the unique countries that have its own alphabet which its citizens are using in its national language (Amharic) and in the form of other regional languages such as Tigrigna. Such uniqueness of the country could have been strengthened and more appreciated if it was possible to use our Ge'ez numbers in all our development endeavors as we are doing in the language. Of course, from the Ethiopian ancient history, it is believed that among the first peoples to inhabit was the Ge'ez speaking agrarians who settled in the Tigrayan highlands around 2000BC. The inland Axum Kingdom was founded by Menilik I, a descendent of King Solomon and the Queen of Sheba, after the fall of the Da'amat Kingdom. Even though, Christianity was introduced into Ethiopia during the Axum kingdom in the fourth century, King Ezana of Axum made Christianity the official religion around 700AD. When the Kingdom of Axum become weak due to the expansion of Muslims, it is replaced by the Zagwe' dynasty between 1137 and 1270 where its significant contribution was the creation of eleven churches carved out of stone, which continued to stand until the present time in the city of Roha (Lalibela, a historical place registered in UNESCO).

As mentioned above, Ethiopia has its own and unique alphabets which become a source of different Ethiopian languages like Amharic and Tigrigna. Associated with the letters in the Ethiopian alphabet starting from ancient times, there are the *Ge'ez numerals* also called *Ethiopic numerals*. At this point it could be reasonable to mention the concept of "*Ethnomathematics*", (even though

ethnomathematics is a wide concept and not the focus of this article), related to this topic which was described by D'Ambrosio (1985) at the International Congress of Mathematics Education (ICME) in 1984. In this regard, D'Ambrosio has explained ethnomathematics by considering three components termed as “**Ethno**” meaning *people*, “**mathema**” to mean *understanding* and “**tics**” referring to *techniques*. Hence, according to D'Ambrosio, ‘*ethnomathematics*’ refers to the culturally equipped methods and approaches of understanding the mathematical procedures by the human society who practiced and shared variety of mathematical concepts and methods in their way of living. In general, ethnomathematics can play the role of a connecting bridge between preservation of culture and local knowledge (wisdom) with the technological and artistic advances through scientific approaches.

Here, there are the following controversial issues:

On the one hand, for many years our experiences and practices show that the use of Ge'ez numerals has remained restricted mainly to the church. These numerals appear on the bible and mainly the church personnel use them. It is also observed that these numerals did not develop in line with the current development of science and technology in general and mathematics in particular as can be seen throughout the world.

On the other hand, from the ancient Ethiopian civilization we observe the construction of the Axumite Obelisks, the castle of Gondar, the churches of Lalibela and many others which should have been demanded high level of engineering/architecture and mathematical knowledge. It could be obvious that such constructions at that time were guided by the religious people and yet the knowledge of mathematics required to build such complex constructions is unquestionable. At least, to attain such perfections in these constructions, people should have been measuring using some sort of mathematics. Therefore, those people who were existed during the time of the ancient Ethiopian civilization should have been using numbers (mathematics), for example, to count and audit their soldiers, military logistics, ship and boats, cattle, and other properties.

The Curriculum Framework for Ethiopian Education document, MOE (2009) underlined the importance of ‘Respect cultural heritage and diversity’ and stated that ‘Ethiopia has diverse cultures that all contribute to the colorful tapestry, which is our country. Young people will be educated in a way that respects this diversity while unifying them into one country.’

Thus, this short research work tries to address these problems.

1.1. Literature Review

The Ethiopic writing system in general is a system created and progressively upgraded in a highly diversified ecological and human environment. The exact date of its origin is not yet known. The fabrication of the south Arabian or external paradigm in the 19th century further undermined the effort to accurately determine the historical origin of the Ethiopic Writing system (Ayele Bekele, 1997). In his work Ayele (1997) further addresses that the Ethiopic syllabic writing system is a writing system with the following major properties: pictography, ideography, astrographs, number system, grammar and syllabary. The system is defined, therefore, as a system of knowledge and knowing through concretized symbols. It is also a matrix of knowledge for philosophy, religion, astronomy, and linguistics. Of course, there is a tradition of replicating manuscripts on new *birana* (i.e. writing pad made up of leather) after a given period of time. And the practice of replication makes dating the original script or manuscript very difficult. But the inscriptions on monuments from northern Ethiopia suggest that the system is at least 3000 years old. Most existing literary manuscripts, however, are dated from 12th century of this era. More in detail, it is explained in Ayele (1997) that the inscriptions that are primarily inscribed in Ge'ez or first column syllographs (that is, Ha to Pa) provide royal biblical references that have been dated starting from 1000 BC. For instance, the ideographic names of the 26 main syllographs are sources of symbolic conceptions, material interpretations, regional identification, holistic paradigm and notion of order. Furthermore, the 182 (that is $7 \times 26 = 182$) syllographs of the system not only have numerical value, but they are linked to texts in the *Old* and *New Testaments* as well as other biblical sources.

The astrographic features of the syllabic table constitutes another level of knowledge. The system can be viewed as a classic example of holistic paradigm or structure of knowledge. The styles (ordinary to calligraphy) and the levels of progression of the system are also good markers of historical time and place (Ayele B., 1997).

Clearly, the ancient Ethiopian civilization which lasted relatively for a long time and the development of Ethiopian (local) mathematics, and science and technology in general are not compatible. The development and/or decline of science and technology in general and mathematics, based on Ge'ez numbers, have been influenced by a number of factors. For a long period, church education which was highly dominated by religious beliefs was the only means of educating citizens. Of course, it is not more than a century since modern education is introduced to Ethiopia. The major issue that can be raised here is the socio-cultural beliefs of those individuals and institutions of the country which, in a way, could influence the advancement of mathematics in the country. D'Ambrosio (2001) argues that

“The distinct ways of doing (practice) and knowledge (theory) that characterize a culture are part of the shared knowledge and the behavior that has become compatible.”

Thus, the socio-cultural issues may include traditional beliefs, religious beliefs and views towards political power and security. Ethiopia, as a country with rich natural resources and source of Blue Nile, was a place where many states of the world put their

attention. In this regard, history tells us that, on the one hand, Ethiopia has been facing long-lasting wars with external invaders and internal oppositions against the existing states.

On the other hand, as the governance system of the country was highly dominated by religion, in defending external invaders there was the belief of keeping what people know as secret, and through time, I think, this became a tradition. Thus, there was neither generation of new knowledge nor transfer of the existing knowledge to the young. Therefore, such a culture has badly influenced mainly the development of the Ge'ez numeration system leading to the domination of our own numbers by those of foreigners. For example, in the twelfth century Alexandria appointed an Egyptian archbishop to Ethiopia, whose title was *abuna*, meaning "our father." The bishop appointed was always of Egyptian origin and it was not until 1950 that a native Ethiopian was appointed the position of *abuna*. This process, obviously, had its impact on most issues of the state including education in general and the development of mathematics in particular. The point, here, is it would be very important to deeply analyze the implications of socio-cultural and traditional beliefs and principles towards sharing nationwide issues such as the origin, development, status, and future challenges and opportunities of the Ge'ez numerals. To this end, the analysis of evidence collected regarding these socio-cultural issues is guided by the interpretive approach to research. Of course, most qualitative research emerges from the interpretivist paradigm. As stated by Creswell (2007), qualitative researchers use these paradigms to shape the types of questions and problems examined, approaches to data collection, analysis, evaluation; and the use of information to change society. Even though, interpretivist views have different origins in different disciplines like anthropology and sociology, as a methodology they generally share the following points:

- Interpretive approaches rely heavily on naturalistic methods- interviewing, observation, and analysis of existing texts.
- These methods ensure an adequate dialogue between the researchers and those with whom they interact to collaboratively construct a meaningful reality.
- Meanings are emergent from the research process.

1.1.1. The “A-be-ge-de” versus “He-Le-Ha-Me” Arguments

Ethiopia is one of the oldest countries in Africa; the emergence of Ethiopian civilization dates back thousands of years (https://en.wikipedia.org/wiki/History_of_Ethiopia).

The modern Ethiopian education system during the different past regimes did not fully reflect the people’s social, economic, and cultural conditions. This is, mainly, because of the country’s curriculum is either directly copied or developed with the involvement of Western expertise (Nair and Abera, 2017).

However, there are indications that the earliest known inscriptions in the Ge'ez or Ethiopic script date back to the 5th century BC have been possibly developed from the *Sabaeen* script where the type of writing system was “**Aa-Bu-Gi-Da**” and the writing direction is from left to right in horizontal lines. In modern communities that are using Ge'ez, such as in the Amharic and Tigrigna languages, it is called ‘*FIDEL*’ which means “*script*” or “*alphabet*”.

Even though adequate sources are not available to generalize, along with the Ge'ez scripts it seems that the simplest form of enumeration of Ge'ez numerals applies only to the first form letters arranged in the Hebrew or Greek order (retrieved from www.geez.org/Numerals/Numerology.html).

Thus, the Ge'ez ‘**Abegede...**’ order corresponds, for example, to the Greek ‘**ΑΒΓΔΕ...**: Alpha, Beta, Gamma, Delta, Epsilon...’ representing numerical values one, two, three, four, five, etc. in both the Greek and Ge'ez cases. It is also well known that Ethiopia has the historical symbols that are still used for counting, for example from 1 up to 10 and to count multiples of 10 such as 20, 30, 40, ..., 100 and so forth (Denbel, 2023). More specifically, as presented by Solomon, A. (2005), the details of the commonly known Greek system numbers are listed as follows:

α	β	γ	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	ξ	ο	π	ρ	τ	υ	φ	χ	ψ	ω	τ
1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	100	200			

Thus, there are arguments that these numerals have developed from Greek alphabet, possibly via Coptic and such assignment of letters for numbers go up to the Greek Ω (Omega) and the corresponding Ge'ez letter Psa (Τ) both representing 800. Some scholars also tried to relate, particularly, the bars above and below the letter-like symbols appear reminiscent of their Roman counterparts. But, the symbols in between the bars are clearly not of Roman origin. This shows that the origin and development of Ge'ez numerals may be far beyond the introduction of Christianity to Ethiopia. Some sources also indicate that enumeration of letters to serve as numbers for Semitic scripts was common (retrieved from www.geez.org/Numerals/Numerology.html).

Since the applicability of the ‘**A-be-ge-de**’ enumeration system was not clear or not well understood, it may be a scholarly alternative to the preferred ‘**He-le-ha-me**’ system of the ‘*Debtera*’ enumeration. A *Debtera* is an adjunct member of the Ethiopian Orthodox church who is expected to have a wider range of learned skills than what is required of a priest and perform the music and dance associated with church services and also sometimes function as astrologists, wizards, fortune tellers, and learned church lore. It is expected, therefore, that this system (that is, proponents of the ‘**Ha-Le-Ha-Me**’ system) extends the enumeration for the original seven syllabic forms of the alphabet and they testify to the better results that the system offers when applied as *Gametria*-a word

derived from the Greek word “*geometry*” also known as “*sacred geometry*” which is the practice of assigning numeric value to letters to reveal hidden meanings and relationships through the summation of the letters in keyword found in sacred text.

A basic example use of the enumeration system applied to fortune telling by the Debtera is to take a person's mother's name, find its summation from the first form characters, take the modulo of twelve and compare it against the corresponding star for the month. The resulting star indicates what fortune that person can expect in his/her lifetime. This use of numerology with astrology is in the much the same tradition as, for example, the monthly Zodiac system still popular in the west. However, this class of character information could not widely be applicable to text and information processing in this modern world. That is, Ethiopic numerology still in use mainly plays a role in modern society analogous to a ‘Zodiac’ type fortune telling system. The point that can also be raised is whether there was mathematics in Ge'ez numbers for other purposes other than mentioned above. Regarding this question Yitbarek (2009) gives more of religious explanation. He accepts the presence of mathematics and mathematical operations starting from the very beginning of the human creature by associating addition with wealth, longer life, respect, love, peace, and wisdom; subtraction with lack of respect, dispute, poverty, stress, and death; multiplication with population increase and reproduction through the good will of God; and division with equality, sharing, and/or personal ownership (pp. 54-55). In my view, this could only be thought of as more of personal and religious beliefs than related to mathematics. But I can argue that people were at least counting their people, military logistics, and other properties thereby dealing with increment (addition), decrease or reduction (subtraction), double of something (multiplication), or half of something (division), and so forth. The problem is that such thinking and usage of numbers during ancient times is not well documented or not found.

On the other hand, particularly the introduction of Christianity to the Ethiopian society has contributed a lot of positive things, such as for example, preserving cultures and natural resources, developing education and traditional medicines. In this regard, Eric Appau Asante, Stephen Ababio, and Kwadwo Boakye Boadu (2017) argue that

“Indigenous cultural practices play a significant role in the conservation of forests. Most of the forests within traditional communities in the Ashanti region of Ghana were preserved centuries ago through traditional beliefs and practices.”

Similarly, everyone can also observe that the same preservation of forests is happening throughout Ethiopia where church sites are located.

However, one can observe that there was a problem of consistency in the development process and application of the Ethiopic numbers. Whatever the case may be Yitbarek (1994) states that our Ge'ez numbers are becoming dominated continuously by the Hindu-Arabic (now understood as European) numbers. For this reason, the Ethiopian young generation is now almost ignorant of these numbers and hence the best solution is to do mathematics using the Ge'ez numbers (P.1).

1.2. Is there any additional problem?

Another issue that may be raised in relation to Ge'ez numerals is about the number zero (Albo). Of course, the Ethiopic numeral system is one of the ancient alphabets that did not have the zero number. Therefore, it may be possible that the absence of zero necessitated the creation of different sets of numbers above nine in ancient character sets like that of the ‘**Abegede**’ system or the ‘**Halehame**’ system.

Dr Aberra Molla (1997) retrieved at <http://www.ethiopic.com/ethiopic/numerals.htm>, addresses the problem by stating that the difficulty of accepting zero, a number that did not represent something to count, by past civilizations may be understandable; but the absence of zero might have also made it difficult to comprehend negative numbers. For example, the Amharic typewriter has only the Arabic numerals and the exclusion of the Ethiopic numbers may have been because of the lack of Ge'ez zero and the subsequent mechanical difficulties to include zero less digits. Generally, there is nothing wrong with the Ethiopic digits one to nine though some could use improvements like the lower or upper bars, or both could be removed to reduce strokes in writing. But Ge'ez (Ethiopic) zero should be introduced and utilized to avoid confusion and to correct ancient mistakes (if any). Obviously, it is also difficult to calculate with a number system without zero since there is no possibility of denoting zero, place value, decimal or negative values. For example, Amdework Mitiku (2000) on his short communications paper to the journal of the Society of Ethiopian Electrical Engineers (SEEE JEE) retrieved from www.yebbo.com/sydneyolympic/MARATHON/.../zero_at_SEEE.doc, states that “given the powerful impact of binary digits, one could not help wondering how long it would be before the Amharic/Ge'ez numeric system stopped borrowing numbers from the English language for mathematics, science, engineering, and several other applications. There is no way to apply this marvelous concept of digital electronics to the Amharic Ge'ez numeric system, unless one borrows a symbol for ‘0’ from another number system. But that was what exactly one sees in Amharic newspapers, personal letters, magazines and several works of literature.”

Amdework (2000) further addresses his concern on the importance of zero in the Ge'ez numerals by stating that “...if there is no zero (0), there will be no digital application. So, how can the Amharic/Ge'ez numeric system participate in an increasingly digital world?” Of course, a symbol for the Ge'ez zero is proposed by some scholars which looks like θ with the justification that it is closest to the symbol used for “empty” or “null” and that will not be difficult to remember. Azeb Amha (2010) recognizes that the attempts made by Kidanewold and Amdework to introduce a grapheme for zero but this and other reform initiatives did not acquire wider usage due to many reasons. Yitbarek (1994, 2009) on his part justified that the symbol for the Ge'ez hundred without the

upper and lower bars or the Amharic letter “Ye” (የ) should play the role of Ge'ez zero (*Albo*, meaning ‘*nothing*’ in Tigrigna). But still consensus is not reached on such zero digits and hence the appropriateness and applicability of this new Ge'ez zero is not clear. So, these and other gaps on the development and applicability of the Ge'ez numeration system may be addressed by extensive research.

2. Methodology

The general approach employed in this study is narrative research design.

Creswell (2007) defined this approach as ‘Narrative is understood as a spoken or written text giving an account of an event/action or series of events/actions, chronologically connected’. Thus, I felt that the research topic can better be studied by this narrative research design which considers the following important questions as put by Creswell (2007):

- Who owns the story? Who can tell it?
- Whose version is convincing? and
- What happens when narratives compete?

2.1. Data Type and Source

Since the study depends on historical areas, the data type collected was purely qualitative data.

The sources of data for this study were both primary and secondary. The primary data was collected directly from informants with the help of interview. The potential sources of information or informants were purposely selected to receive rich data. The secondary data were collected from different documents (eg. books, magazines), manuscripts, records, artifacts, etc.

2.2. Data Collection Methods

As this study is mainly qualitative research (employs narrative strategy) in nature, interviewing and document analysis were the major data collection methods.

2.2.1. Interviews

2.2.1.1. In-depth Interview

This is conducted with monastery administrators, priests, administrators and experts of historical and cultural sites or offices, with purposely selected elderly and knowledgeable individuals, etc.

2.2.1.2. Focus Group Discussion (FGD)

Focus group discussions were held (conducted) with group of teachers, students, and administrators of the Theology college of Addis Ababa. In such focus group discussions, three to eight participants have been formally formed by the investigator.

2.2.2. Document Analysis

As possible sources of information for the study, available books, manuscripts, field notes from the different sites have been handled during data collection process. Other documents from different libraries like the Ethiopian Collection at The Kennedy Library of Addis Ababa University have been considered.

2.3. Data Analysis Approach

The story obtained from interviewing people and from documents are narrated and organized. I tried to identify and categorize patterns and finally interpretations and implications of findings are also included in the analysis. Major findings are given in the form of conclusions and recommendations.

3. Analysis of Findings and Discussion

The findings of this short study are presented below, even though not comprehensive as the study is limited to evidences obtained only from a specific area.

3.1. When did Ge'ez numbers used in Ethiopia?

As one limitation of Menilik I, Kidanewold (1956) says that when Menilik I encourage the translation of books brought from Jerusalem during his period from Greek to Ge'ez language, similar attention was not given to the Ge'ez numbers. Yitbarek (1994) as one of his findings from interviewing people, observations, and documents states that language, mathematics, and symbols (pictures) are created or discovered at the same time and together. For example, the writer says that there are Ge'ez numbers written during the Axumite king of Ezana (p. 1-2).

But still there are controversies regarding the actual period where these numerals are used by people and what their origin is. On the one hand, there is no concrete evidence showing that there were such numbers beyond ten thousand (Elif). On the other hand,

Yitbarek (1994) says that there are many scholars who tell us orally that there was a high level of mathematics and the application of mathematics during the ancient time in Ethiopia.

To get more evidence on the issues raised above, I conducted an interview. Of course, after a very lengthy effort I became successful in meeting three resourceful individuals for interview. Three of them appointed me at the same time. In the interview, I used *pseudonyms* and presented the discussion made with the three participants as follows.

Background of participants:

1. Mr. BM: A known teacher in the churches of Addis Ababa with the title of "Like Hiruyan".
2. Mr. ZD: A theology scholar and known teacher in the churches of Addis Ababa.
3. Mr. GM: A university lecturer with MA degree in Philology.

After thanking for their being volunteer to come for the interview, I explained about the current and future purpose of my study and hence the interview. Then, we proceeded with our discussion.

Abreha (Researcher): Would you please explain what you know about Ge'ez numbers, mainly, about their origin and development?

Mr. BM: "I would like to appreciate your initiative to study on this agenda. Even though, there are some individuals who tried to write and publish books related to Ge'ez numbers, their effort was not supported by systematic and scientific research methodology. So, this is a good idea. Now, coming back to your question, most of the Ge'ez numbers are taken from or represented directly by our Ge'ez letters (alphabet)."

Abreha: Can you mention some of the letters that directly represent Ge'ez numbers?

Mr. BM: "Yes, for example, 4 = **፩** (Aa), 6 = **፮** (Ge), 7 = **፯**(Gi), 40 = **፳፱** (Sa), 50 = **፷** (Ha), 70 = **፸** (Ro), 90 = **፹** (The sixth letter in the "Ne" row), and 100 = **፲** (Ye)."

Abreha: So, what does this imply? What is your view?

Mr. BM: "Well, my idea is that most of the numbers are originally taken from our Ge'ez letters.

Many scholars also say that our letters were directly serving as numbers as well."

This time, Mr. ZD interrupted and reflected on this issue.

Mr. ZD: "But, it is also said that these numbers are not of Ge'ez origin. They can also be Roman origin."

Abreha: Is there any justification for being Roman origin?

Mr. ZD: "Yes, because during the ancient Ethiopian civilization there was religious and commercial relationship between Ethiopia and Rome. Thus, there could be some Roman symbols in the Ge'ez numbers adapted from that of Romans."

Mr. BM: "Rather, if not all of these numbers are of Ge'ez origin, most people and the church scholars argue that these numbers are derived mainly from Greek and probably also from Coptic."

Abreha: What is the reason for such argument?

Mr. BM: "Well, Aba Selama Kesatebirhan who came to Ethiopia from Greek at his early childhood and later became 'Abun' of Axum was the one who translated many books from Greek to Ge'ez. Of course, Greek was an international language at that time."

Abreha: So, what does this mean?

Mr. BM: "Until then, it is believed that the original Ge'ez letters in the order of Aa, Be, Ge, De, ..., Pe were serving as numbers in this order as 1, 2, 3, 4, ..., 800, respectively. When Aba Selama put his influence and changed the alphabetical order into Ha, Le, He, Me, ..., Pe; some Greek symbols should have been also introduced into the Ge'ez numbers. Of course, there are many high-level church scholars who are still criticizing Aba Selama for the changes he made in the order of the Ge'ez alphabet."

Mr. GM: "I think most of the numbers are associated with the introduction of Christianity to Ethiopia. So, instead of completely arguing that the numbers have origins other than the Ge'ez alphabet itself, it is good to think that there could also be parallel development."

Mr. ZD: "My belief is that these numbers could have had mixed symbols from Ge'ez, Greek, and Romans. So, this requires further research."

Mr. GM: "By the way, Aba Selama was only coordinating the high-level church personnel in Axum and he cannot make the changes himself alone. So, whether the symbols are borrowed from others or not, they are ours."

Here, it is important to note that there is no any additional indication to support that the Ge'ez (Ethiopic) numbers can be of Roman origin. However, some of the similarities between the Greek and Ge'ez alphabet and their order as indicated in the literature section could imply that some symbols should have been taken from Greek to represent numbers. To continue with the discussion, I raised another question on whether or not there was/is common understanding on the letter-like numbers (symbols) and their meaning.

Mr. BM: "Since people during the ancient time, in my view, were counting using only their alphabet, there were no numbers beyond 800 (**፳፱**). Later, they proceeded up to 'Elif'. Even at present there is no known symbol beyond 'Elif' or (**፳፱**)."

Abreha: What does 'Elif' represent?

Mr. ZD: "Originally, it was understood as 'one thousand', but nowadays there is agreement almost among all church people to use it equivalent to 'ten thousand' or (10,000)." This idea is supported also by Mr. BM.

Mr. GM: "Through time there could also be change and hence flexibility."

Abreha: What do you mean by flexibility?

Mr. GM: "I mean improvement or development. For example, when we see the change from the Abegede order to the Helehme order, this obviously shows some kind of flexibility."

Abreha: What is your view concerning the introduction of a new zero symbol into the Ge'ez number system?

Mr. BM: "Of course, a respected teacher of the church and artist Yitbarek Gessesse Mera has proposed the Amharic letter 'ᐆ' (Ye) or the Ge'ez representation for 'hundred' without the lower and upper bars to be the Ge'ez zero. But, this has also another side effect."

Abreha: What is the problem with it?

Mr. BM: "If it is used as zero, then it leads to distortion on the other numbers."

Abreha: How?

Mr. BM: "Because the original single symbols for twenty, thirty, forty, and so forth will be changed to two followed by zero, three followed by zero, four followed by zero, respectively. This means that we are eliminating our original numbers or symbols."

Mr. ZD: "There is also another clash with using it as zero. Because this same symbol, without the lower and upper bars, in some manuscripts is also used as 'hundred'. So, if it is changed to zero, then the generation to come may understand such manuscripts involving this symbol as zero instead of hundred."

Mr. GM: "Since the Amharic letter 'Ye' and the symbol for hundred are very similar, then there could also be problem in using them in the optical character reader (OCR)."

Abreha: Then, what is your proposal so that the Ge'ez numbers can be applied in arithmetic?

For this question, all respondents said that there is a need to conduct detailed research on the issue. For the purpose of using computer technology, according to the participants, the Arabic zero could also be adapted into the Ge'ez numeration system.

Abreha: What is your reflection on the use or trial of Ge'ez numbers at school level?

Mr. BM: "So far, the major application of these numbers that we are observing is mainly limited to the preparation of the Ethiopian yearly calendar. However, if the problem of the absence of zero and other related issues can be solved, I think it is possible to introduce arithmetic in schools. But this should not be addressed in a very complex way. It should be designed in a very simplified manner so that people can be attracted and easily understand the concepts related to operations with Ge'ez numbers."

Abreha: Do you have any general comments?

The participants underlined that as this is an important national agenda and a potential research area, the study should be conducted at large scale. They also expressed their commitment to provide any support they can.

From this discussion, the dominant idea about the origin of Ge'ez numbers is related with those of Greeks. But this may not be taken as concrete evidence that they are of Greek origin. There is no clarity whether we have borrowed or the Greeks have borrowed from Ge'ez.

3.2. The Existing and Missing Elements in the Ge'ez Numeration System

Knowledge is not an entity which can be collected as in knowledge acquisition models. Instead, knowing is reciprocally co-determined between the agent and environment. The reciprocal interaction cannot be separated from the context and its cultural and historical constructions (Lave, 1988). In some societies including Ethiopia, indigenous mathematical knowledge is partially or totally hidden (see Tables 1 and 2) and hence there is a need to try to 'uncover' this 'hidden mathematics' (Gerdes, 2005). In this regard, Yitbarek Gessesse (2009) in his 3rd book written mainly about Ge'ez numbers in Amharic argues that Axum is the source of Ge'ez numbers, which these numbers have been practiced for a long time together with the introduction of the Ge'ez languages in Ethiopia in general and in Axum (Tigray-Northern Ethiopia) in particular. Different evidence indicate that the Ge'ez numbers are found and used in different books, magazines, newspapers, and Ethiopian currencies (coins and birr notes). Especially in the old curriculum, most elementary textbooks used these numbers but neither the teachers nor the students know what actually the numbers are representing. In addition to this, in the entire Ethiopian birr notes we see both the European and Ge'ez numbers and almost all citizens, except probably the priests (because they know only the Ge'ez numbers); give no conscious attention to the Ge'ez numbers on the notes. From the experience of my own and many others, for example, the 'one thousand' in Ge'ez numeration system occurs either by writing the Ge'ez symbol for number 'one' followed by the Amharic word 'thousand' (ᐆᐆ) or by putting the Ge'ez numeric symbols for 'ten' and 'hundred' together (ᐆᐆ) in this order which is read as 'ten hundred'. It is not clear whether the Amharic letter 'Shi' (ሻ) is playing the role of a number or a letter. These different representations for the same number, obviously, show that there is inconsistent use of the symbolic representation of the numerals. In the latter expression, it seems that the symbol for the 'hundred' plays the role of zero or double zero. Another example which leads to a serious confusion and difficulty in understanding the quantitative meaning of these numbers is the use of Ethiopian calendar such as 2000 E.C (Ethiopian Calendar)

which is sometimes written by putting the Ge'ez numbers for 'twenty' followed by 'hundred' (፳፱) which is usually read as twenty hundred (which seems 20 times 100). In some other cases, an expression to 1996 E.C in Ge'ez numbers is found involving the symbols for 'ten', 'nine', 'hundred', 'ninety', and 'six' in this order (፲፱፻፲) which is again usually read as 'nineteen hundred and ninety-six'. If the symbol for hundred was to represent the Arabic number 100 instead of zero, then this number would have been read or understood as 190096 or even 1090096. So, in trying to translate these symbols, there is no consistent approach. In general, evidence show that there is no common understanding on the quantitative meaning of these 'letter- like' Ge'ez numeric symbols. According to the findings by Yitbarek (2009) from interviewing about 44 high level church scholars from all over the Ethiopian monasteries, there was a confusion on the meaning of the double symbol for hundred (፱፱). For example, those church scholars he interviewed in Axum understand the double symbol for the Ge'ez hundred (፱፱) as thousand. On the other hand, this symbol in most cases after the year 1948 is changed to or understood as ten thousand (p.47) which means 100 times 100. Again, some interviewees of this report by Yitbarek responded that they know terms for large Ge'ez numbers like *Elif* (ten thousand), *Aelaf* (hundred thousand), *Aeilafat* (one million), *Mielifit* (hundred million), *Tielifit* (one billion), and many more to mean "many beyond the imagination of the human mind" and hence not representing some numerical value (2009, p.37). The inconsistent use and misunderstanding can be seen in different dimensions. For example, in his book 'Metshafe Sewasew', Tesfa G.Selassie (1990) starts at 'ten thousand' (*Elif*) in listing some large numbers without addressing 'one thousand' and proposes as indicated in the following Ge'ez numeration hierarchy (see numerals Table 1):

Table.1. Sample numeration hierarchy

Ge'ez numeric symbol	Ge'ez Name	Translation	Numerical value
፱፱	Elif	Ten thousand	10,000
፲፱፱	Aelaf	Hundred thousand	100,000
፱፱፱	Aeilafat	One million	1,000,000
፲፱፱፱	Tielifit	Ten million	10,000,000
፱፱፱፱	Mielifit	Hundred million	100,000,000

Thus, in many documents and books like the one mentioned above, some components such as ten Elif, hundred Elif, ten Aelaf, hundred Aelaf, ten Aeilafat, hundred Aeilafat, etc. are missing from the expected Ge'ez numeration hierarchy. According to Yitbarek,(2009) the change of the double symbol for hundred (፱፱) from 1000 (one thousand) to 10,000 (ten thousand) is supported by the decision or recommendation made by a committee established by the Addis Ababa church council to respond on this issue (2009, P.41). I think this is a step forward towards creating a common understanding of the large Ge'ez numbers and to further progress on the overall numeration system. From the different sources analyzed so far, it could be possible to put the following more comprehensive and relatively acceptable hierarchy on the Ge'ez numbers.

The comparison is made with the American system (Yitbarek, 1994 & 2009). According to the hierarchy indicated in table 2, one can observe that while the European large numbers are separated by comma at every three digits starting from the unit's digit, the Ge'ez numbers are separated by colon (instead of a comma) at every four digits. Now, from what I tried to describe above, one can understand that the Ge'ez or Ethiopic numbers exist and to some extents are in use in the bible, calendars, books, posters, birr notes, and magazines. For example, from my personal observations, I came to understand that there are individual initiatives in trying to develop computer software programs using Ge'ez numbers in preparing annual Ethiopian calendars.

Table.2. Additional Ge'ez numbers and their equivalent translations

Ge'ez Name	Translation	Numerical value	No. of zeros
One Elif	Ten thousand	10,000	4
Ten Elif	Hundred thousand	100,000	5
Hundred Elif	One million	1,000,000	6
Ten hundred Elif	Ten million	10,000,000	7
One Aelaf	Hundred million	100,000,000	8
Ten Aelaf	One billion	10 ⁹	9
Hundred Aelaf	Ten billion	10 ¹⁰	10
Ten hundred Aelaf	Hundred billion	10 ¹¹	11
One Aeilafat	One trillion	10 ¹²	12
Ten Aeilafat	Ten trillion	10 ¹³	13
One hundred Aeilafat	Hundred trillion	10 ¹⁴	14
Ten hundred Aeilafat	One Quadrillion	10 ¹⁵	15
One Eielif (E'Ye'Li'Fat)	Ten quadrillion	10 ¹⁶	16

Ten Eielifat	Hundred quadrillion	10^{17}	17
Hundred Eielifat	One quintillion	10^{18}	18
Ten hundred Eielifat	Ten quintillion	10^{19}	19
One Eielifat (E'Li'Fi' Tat)	Hundred quintillion	10^{20}	20
Ten Eielifat	Sextillion	10^{21}	21
....
Tielaf (Te'E'Laf)	Septillion		
One hundred Mielifat (Mi'E'Li'Fi'Tat)	Sept decillion		

But, generally, the appearance of symbols, their translation and meaning has a lot of misunderstanding and inconsistencies. In my opinion, all such differences discussed above show that no attention was given to the Ethiopic numbers and their application by the concerned individuals and institutions like curriculum developers and experts, the education sector officials, school teachers and administrators, church scholars, and other stake holders.

3.3. There is a Need for Modification

Yitbarek (1994) proposes the need for modification of the Ge'ez numbers by removing the bars above and below the original skeleton of the numbers so that doing mathematics with the becomes easier. The reason for such removal is because these bars take more space, we waste time when we do mathematical operations with them, and they are not comfortable for performing mathematical operations, but such modification does not; basically, change the nature and value of the numbers (p. 20). Beyond this modification, evidence indicate that the Ge'ez numbers cannot be used for doing arithmetic and other mathematical operations without the introduction of Ge'ez zero. For example, without the Ge'ez zero it is impossible to deal with place value, to study about decimals and fractions, with the four commonly used operations (addition, subtraction, multiplication, and division) and higher mathematics in general. Of course, the double symbol for the Ge'ez or Ethiopic hundred nowadays is mostly understood as $100 \times 100 = 10,000$. Also, if someone who knows something (is familiar) about these numbers is asked to add 'twenty' and 'fifteen', then s/he can give the correct answer which is 'thirty-five'. However, 'twenty' is a single symbol and 'fifteen' has two symbols. And since there is no the idea of units, tens, hundreds, and other place value positions, such addition cannot be performed in written form as we do in the Arabic numbers. In addition, people in their day-to-day activity usually talk and operate with fractions such as half, one-third, one-fourth, two-third and so forth, but all these concepts cannot be transformed or translated into written documents and hence they are only understood implicitly and expressed orally. Even though there is a need for further research since consensus is not reached among scholars and different stake holders about these issues, Yitbarek (1994) proposes the Amharic letter "Ye" (የ) or the Ge'ez hundred without the bars above and below to play the role of zero. The justification for this which seems, in my view at this level, is acceptable is that this symbol in the original numbers indicates multiplication by 10, by 100, by 1000, and in general by powers of 10. Accordingly, all Ge'ez numbers can be used for doing mathematical tasks without any problem. If all the above modifications are made, therefore, the original twenty and above Ge'ez numbers (these are mostly single letter-like symbols) shall be used for the traditional use such as page number of books, calendars, and other related issues and not for doing mathematical operations.

4. Conclusion and Recommendations

From the analysis of the evidences presented so far, the following preliminary conclusions and recommendations can be made.

4.1. Conclusion

First, the Ge'ez or Ethiopic numbers are not fully forgotten. From this limited study, it can be concluded that people were using and are still using Ge'ez numbers in Ethiopia. Most of the priests or church personnel who can read and write know only these numbers and not the others. These numbers are widely found in the bible and other books as page, chapter or unit numbers. They are also used in all the Ethiopian coins and birr notes. Generally, even if the continuous development of these numbers may be thought as interrupted at some point in the past, as a revival of such development, some individuals have also tried to develop converter software that is used to convert calendars from European system in to Ge'ez and vice versa. I think, this is helpful to at least sustain the Ge'ez numbers.

Second, there is lack of common understanding on the meaning. The study indicated that symbols representing Ge'ez numbers are understood differently by different people including the high-level church people. For example, there is no full agreement on the translated meaning of 1000 and 10,000. Another misunderstanding and confusion, for example, is the symbolic representation for 'two thousand and four' (eg. 2004 E.C.) or ፳፻፬ in the Ethiopian calendar confuses with 'twenty hundred and four' which may be translated as 20,004.

Third, there is no agreed zero, place value, and mathematical operations (eg. Addition, subtraction, multiplication and division). Even though the Ge'ez numeric symbol for hundred in some cases seems to play the role of zero, there is no known and accepted zero in this numeration system. This, in turn, leads to lack of systematically established number system. Thus, there no place value,

no well-established arithmetic, and in general there is no any mathematical operation in practice. Therefore, there is a need for the overall modification of the numeration system, including the introduction of Ge'ez zero so that we can use them. It is important to raise the question 'Is it not possible to introduce a Ge'ez zero?', and try to answer it. If we cannot use these numbers in a better- and well-established way, then the coming generation will be challenged even to read manuscripts involving numbers and understand its own history. Thus, if emphasis is given to these Ge'ez numerals with the agreed Ge'ez zero (Albo) number and at least the four arithmetic operations (Addition, subtraction, multiplication and division), then children studying the primary level curriculum can play, learn, and develop this indigenous knowledge and this knowledge can support also for developing deep mathematics understanding. To summarize, this knowledge can 1) Promote students' higher level thinking capacity, 2) Improve students' societal problem-solving strategies and creativity, and 3) Develop personal communication skills with local communities, for example, with the priests.

4.2. Recommendations

Development of a country demands multidimensional effort. One of the most important components is education and the effort citizens make towards enhancing it. Since the ancient history and civilization of Ethiopia is found in manuscripts and different documents mainly written in Ge'ez language and Ethiopic numbers, making significant effort on our own scripts and numbers can lead us to rapid and sustainable development of our country. Thus, the following preliminary recommendations may help make this vision a reality.

Awareness Creation and training: Ge'ez numbers are used in different text books and reference materials. However, neither teachers nor students know about the numbers. Thus, it is my recommendation to design a periodical and continuous awareness creation forum and provide short term training for teachers and other interested social groups.

Consensus Building: Consensus shall be built among different stakeholders to come up with common understanding on issues like the importance of sustaining Ge'ez numbers, the meaning of the symbols, the introduction of Ethiopic zero, the need for modification of the numbers, the need for place value, and signs for arithmetic operations. Of course, the letter-like Ge'ez numbers involving bars above and below should be modified and transformed into another (new) version in order to operate or do mathematics using them.

Introducing into school system: In the context of the education reform in Papua New Guinea which strongly encourages the use of both Indigenous knowledge-based systems and local vernaculars in teaching various school subjects, research finding by Rex A Matang reported that

"While many of these curriculum reforms were aimed at pursuing improvements to both teaching and learning of mathematics in schools, they have, however, often ignored the socio-cultural aspects of mathematics education" (p.505)

In view of this argument, even though in some of the Ethiopian colleges and universities, a course on 'number theory' is offered, this course does not address Ge'ez numerals. I recommend, therefore, that it is important to address these numerals, their historical origin and development and/or decline, and current status of the numbers in the course. In addition, Ge'ez as a language is offered at Addis Ababa University, Theology colleges of Addis Ababa and Mekelle-Tigray region, and in at least one of the universities of Germany. Therefore, it seems very important to offer an elementary course on Ethiopic numerals which can be implemented in an integrated way with the Ge'ez language at these institutions. In addition, if interested or selected teachers are aware of the Ge'ez numbers through short-and/or long-term trainings, then it would be possible to design a simple school curriculum on Ge'ez numeration system in the form of chapters or of a course, in collaboration with the Ethiopian Orthodox Church Theology colleges in the country, for beginners and can practically be tested at elementary and/or secondary school level both for teachers and students.

Further research: As the scope of this study is very limited research and hence not exhaustive, then detailed and extensive research need to be conducted on the agenda. If such deep studies are conducted and become successful, then they may be helpful to think and do mathematics using our own numbers as we are doing with our own alphabet and this, of course, does not lead to any contradiction/loss with the modern mathematics. But, using our own numbers or numeration system does not also necessarily mean that we have to withdraw from using the existing numbers or from using the existing mathematics, science and technology that the whole world is using.

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